Partial Objects and Discourse Representation Theory

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Abstract

In this article the DRT rule for introduction of markers in a DRS is interpreted as the logical rule of *existential instantiation*. Consequently, markers are taken to be constants witnessing existential formulas, instead of existentially quantified variables. Models are introduced, where these witnesses denote *partial objects* and various proof systems are studied. Finally, a 'dynamic' extension of the 'static' DRT framework is proposed.

Introduction

In traditional *Discourse Representation Theory* (DRT) the occurrence of the indefinite noun phrase 'a man' in the sentence "A man walks in" gives rise to a twofold action: (1) a *marker* 'x' is introduced in the discourse representation structure (DRS) for future reference to this man, (2) a *condition* "$man(x)$" is added to the DRS to the effect that this marker stands for a man. Semantically the marker is interpreted as an *existentially quantified variable*: the DRS is true with respect to a particular model if *there is* a variable assignment, mapping x to an element in the domain of the model, such that the predicate *man* holds for this element.

Now, a logician can't fail to notice the resemblance between this rule for the introduction of a marker and the logical rule of *existential instantiation*.

$$\frac{\exists x\ man(x)}{man(a)}$$

and note that the introduction of the marker and the addition of a condition constitute a *single* logical action. Under this interpretation a marker is no longer seen as an existentially quantified variable, but as a *witness* for the corresponding existential sentence.

When we interpret markers as witnesses for existential sentences, they are *constants*, and, unlike variables, constants denote. What element of the domain can we take these constants to denote? Every individual we choose will have a lot more properties than are accounted for by the text. In fact, the totality of his properties will constitute a maximally consistent set. Consequently, this interpretation of marker introduction, forces us to consider the denotation of markers as *partial(ly specified) objects.* A partial object a associated with the phrase "a man" will be an object with only those properties that can be concluded from the fact that it is a man. In the models we will construct, this takes the form of the following equivalence:

$$\mathcal{M}^* \models \forall x(man(x) \rightarrow \xi(x)) \iff \mathcal{M}^* \models \exists x\ man(x) \rightarrow \xi(\overline{a}).$$

Notice that this interpretation of partiality gives us one of the so–called 'donkey equivalences'.

1 Models for Partial Objects

The semantic structures of our theory are, in essence, K. Fine's Arbitrary Object Models (Fine [2, 3]). They contain two disjoint domains: a domain M of *standard individuals* and a domain A of *arbitrary objects.* Every arbitrary object in A is associated with a set of individuals from M, the set of its *instances.* This association will be formalized by a set V of (finite) functions from A into M: for $v \in V$, the element $v(a)$ of M is taken to be an *instance* or *completion* of a.

$\mathcal{M}^* = \langle M,\ A,\ V,\ <,\ I\rangle$ is an *Abitrary Object Model,* (AO–Model) if

1. M and A are disjoint sets. M the *standard* domain consisting of standard individuals, A, the *generic* domain, consisting of arbitrary objects.
2. $V \subseteq M^A$ is a set of finite functions from A in M.
3. $< \subseteq A \times A$, the *dependency relation,* is a strict partial order that is conversely well–founded. (More will be said about this relation.)
4. I is an interpretation function, satisfying: for $\overline{c} \in \mathcal{C}$: $I(\overline{c}) \in M$ and for $\overline{a} \in \mathcal{A}$: $I(\overline{a}) \in A$.

Truth

The idea behind the notion of truth regarding sentence containing constants denoting arbitrary objects is that these objects have a certain property if all their completions have that property. This is the principle of *generic attribution.*
Given $\mathcal{M}^* = \langle M,\ A,\ I,\ <,\ V\rangle$, we will assume to have a set $\mathcal{C}$ of constants $\overline{m}$ for all elements of M, and a set $\mathcal{A}$ of constants $\overline{a}$ for all elements of A. The set of constants from $\mathcal{A}$ occurring in a formula ϕ will be denoted by $\mathcal{A}(\phi)$.
The variable assignments g will always be mappings from VAR, the set of individual variables of $\mathcal{L}$, to M; so formulas without constants from $\mathcal{A}$ are interpreted standardly in $\mathcal{M} = \langle M, I\upharpoonright \mathcal{C}\rangle$, the first–order structure underlying $\mathcal{M}^*$. Finally, $v(\phi(\overline{a}_1,\ldots,\overline{a}_n))$ will denote the result of the simultaneous substitution in ϕ of $\overline{m}_i$ for $\overline{a}_i$, where, for $v \in V$, $1 \leq i \leq n$, $v(I(\overline{a}_i)) = m_i$.

Definition 1 Truth Definition.
Let ϕ be an $\mathcal{L}$–formula such that $\mathcal{A}(\phi) = \{\overline{a}_1, \ldots, \overline{a}_n\}$ and let $V_{\mathcal{A}(\phi)} = \{v \in V \mid \mathcal{A}(\phi) \subseteq dom(v)\}$, then

$$\mathcal{M}^* \models \phi\ [g] \text{ iff } \forall v \in V_{\mathcal{A}(\phi)}:\ \mathcal{M} \models v(\phi)\ [g]$$

Definition 2 Validity
For T a set of $\mathcal{L}$–sentences and ϕ an $\mathcal{L}$–sentence,

- the inference $T \models \phi$ is *truth–to–truth valid* in model $\mathcal{M}^*$ if, whenever $\mathcal{M}^* \models T$ then $\mathcal{M}^* \models \phi$
- the inference $T \models \phi$ is *case–to–case valid* in model $\mathcal{M}^*$ if, for all $v \in V$ defined over the constants in T and ϕ, whenever $\mathcal{M} \models v(T)$ then $\mathcal{M} \models v(\phi)$.

Truth–to–truth validity deals with inferences leading from properties of arbitrary objects to other properties. Here the $\mathcal{A}$–constants behave purely as constants. This interpretation does not allow us to conclude the universal validity of $\phi \to \psi$ from the validity of the inference ϕ/ψ. Case–to–case validity of inferences, on the other hand, allows us to *conditionalize*; for finite T: if the inference T/ϕ is case–to case valid, then the sentence $T \to \phi$ is universally valid. In case–to–case validity, the *scope* of the (hidden) quantifier in the premise includes the conclusion. The case–to–case interpretation makes the constants act like *free* variables, where premise and conclusion are evaluated with respect to the same variable assignment.

The Dependency Relation

Markers are introduced at certain points in a discourse, and a new marker may *depend* on previously introduced ones. A representation of the text "A man walks in. His mother follows him as usual" must incorporate the fact that the marker for "His mother" depends on the marker for "A man": if an individual m is a completion of "His mother", then m must be the mother of the completion chosen for "A man". These dependencies between the objects are dynamically created in the course of a text and are sensitive to the *order* in which the sentences are presented. For instance, the text "A woman walks in. As usual she is preceded by her son" will give rise to other dependencies than the ones generated by the above example.
In AO–models, the *dependency relation* formalizes the 'static projection' of these dependencies.
In order to get a general perspective on this relation we will let the constants exhibit dependencies explicitly: whenever a *witness* is introduced for an existential formula, $\exists x\phi(x)$, it will be indexed by the formula that led to its introduction.

$$\frac{\exists x\phi(x, \overline{a}_1 \ldots \overline{a}_n)}{\phi(\overline{a}_{\phi(x,\overline{a}_1 \ldots \overline{a}_n)}, \overline{a}_1 \ldots \overline{a}_n)}$$

Note that the parameters in $\exists x\phi(x)$ will occur in the index $\phi(x)$ of $\overline{a}_{\phi(x)}$. We will now take these indexed constants to refer to *partial objects*. Partial objects will be arbitrary objects, indexed by formulas from $\mathcal{L}(x)$, the set of all $\mathcal{L}$–formulas in which exactly variable x is free.

Definition 3 Partial Object Models
A *Partial Object Model* (PO–Model) is an AO–model $\mathcal{M}^* = \langle M, A_{\mathcal{L}}, V, <, I\rangle$ where

- $A_{\mathcal{L}} = A \times \mathcal{L}(x)$ with A a non-empty set disjoint from M
- $I(\overline{a}_\phi) = a_\phi$ for some $a \in A$
- $\mathcal{M}^* \models \exists x\phi(x) \rightarrow \phi(\overline{a}_\phi)$
- $a_\phi < a_\psi \iff \overline{a}_\phi \prec \overline{a}_\psi$

Remark 1 Notice that the Truth Definition does not mention the dependency relation. This relation is not involved in truth or falsehood of a sentence, but in its *well-formedness*. For instance, if we have a sentence like "another man walks in" in which the noun phrase *another man* depends on the existence of a (previously introduced) marker $\overline{a}_{man}$, then the absence of such a marker will not affect the *truth* of this sentence, but it will cause it to be non wellformed.

2 Proof System

The proof system belonging to partial object models consists of a standard system of rules of natural deduction for the boolean connectives extended by the following *instantial rules*

Instantial Rules

$$(UI)\quad \frac{\forall x\phi(x)}{\phi(\overline{t})} \qquad (EI)\quad \frac{\exists x\phi(x)}{\phi(\overline{a}_\phi)}$$

$$(UG)\quad \frac{\phi(\overline{t})}{\forall x\phi(x)} \qquad (EG)\quad \frac{\phi(\overline{t})}{\exists x\phi(x)}$$

Here, $\overline{a}_\phi \in \mathcal{A}$ and $\overline{t}$ is any $\mathcal{L}$–term free for x in $\phi(x)$. Notice that only in EI the instantial term is indexed by ϕ; in the constant $\overline{a}_\phi$ introduced by EI, a is uniquely associated with $\phi(x)$. So if we have $\overline{a}_\phi$ and $\overline{b}_\phi$, where $\overline{a} \neq \overline{b}$, at most one can be the result of an application of EI.
The absence of an index in the terms of rules EG, UG, and UI reflects the fact that any term can occur in a valid application of these rules.

To be *sound*, a deduction D in the system has to satisfy the following restrictions (adapted from Copi [1])

Definition 4 Derivations
A derivation D is correct if and only if it satisfies:

WF Weak Flagging: no term a_ϕ that results from an application of EI in D, is also an UG instantial term.

IND Independence: in any application of UG, no A–letter occurring in either the conclusion or the suppositions to the inference can be identical to or depend upon the instantial term.

Weak Flagging speaks for itself, it prevents the drawing of universal conclusions from existential premisses. *Independence* extends the usual proviso for UG — that $\bar{t}$ does not occur in any of the undischarged suppositions on which $\phi(\bar{t})$ is based — to cover all constants on which $\bar{t}$ depends: it may be the case that $\bar{t}$ does not occur in the suppositions, but that we have a suppostion $\psi(\bar{a}_\phi)$ where $\bar{a}_\phi \prec \bar{t}$. Independence rules out this case as a foundation for the application of UG. Given these restrictions, ϕ is derivable from premise set T in our proof system if and only if ϕ is derivable from T in any standard axiomatization of first–order logic. Notice that only the *identity* of the constant enters in the formulation of the restrictions on deductions; consequently, as long as we respect these restrictions, it does not matter whether the constants involved are elements of $\mathcal{A}$ or $\mathcal{C}$. The difference between these constants only shows up in the semantics.

2.1 Extended Proof Systems

In the basic proof system the identifiers of the objects are structurally meaningless. The proof of soundness of the deductive system only mentions the *identity* of the constants. As yet, we have no *relations* between partial objects and even, for instance, the objects a_ϕ and $a_{\phi\wedge\phi}$, or $a_{\phi\wedge\psi}$ and $a_{\psi\wedge\phi}$, may lack common instances. This implies that the general system is sensitive to all structural differences between the identifiers, and we can create a *representational system* by formulating rules that relate objects in terms of the identifiers. To bring this into familiar territory we will introduce a relation of 'local consequence' $\approx\subseteq \mathcal{L}(x) \times \mathcal{L}(x)$ between $\mathcal{L}(x)$–formulas, where will say that $\phi(x) \approx \psi(x)$ holds in model $\mathcal{M}^*$ if *the witness for* $\phi(x)$ *satisfies* $\psi(x)$, i.e., $\mathcal{M}^* \models \psi(\bar{a}_\phi)$ where $\bar{a}_\phi$ is the unique constant associated with $\phi(x)$. To take a classic example from non–monotonic logic. Let $P(x)$ stand for the property of being a penguin, $B(x)$ the property of being a bird, and $F(x)$ the property of flying, then we can have $\mathcal{M}^* \models B(\bar{a}_P) \wedge F(\bar{a}_B) \wedge \neg F(\bar{a}_P)$. This does not constitute an inconsistency, it only means that the value ranges of a_P and a_B will be disjoint. So in this set–up inconsistency is not the problem — we do not have to get rid of certain logical principles — the object, on the contrary, is to add principles in order to *relate* objects. So, for instance,

$$\frac{\psi(\bar{a}_\phi) \wedge \xi(\bar{a}_\psi)}{\xi(\bar{a}_\phi)}$$

would give us transitivity of the conditional $\models\!\!\sim$ and inconsistency of the sentence above. We are free to fix this inferential relation in any way we want, by formulating rules relating the properties of objects with different identifiers.

Reflexivity

An important difference between the conditional as defined here, and standard formalizations, comes out when we look at the principle of *reflexivity*, common to most sub–structural logics, i.e., for all $\phi(\overline{a})$

$$\phi(\overline{a}) \models\!\!\sim \phi(\overline{a})$$

This is not a universally valid inference in our system, because $\mathcal{M}^* \models \phi(\overline{a}_\phi)$ only holds in models where $\exists x \phi(x)$ holds. In other words $\phi(\overline{a}_\phi)$ is not an axiom in our system, but the conclusion of a rule.

$$\frac{\exists x \phi(x)}{\phi(\overline{a}) \models\!\!\sim \phi(\overline{a})}$$

Here $\exists x \phi(x)$ gives rise to the introduction of a *reflexivity statement.* What this means is that our proof system will only give us sound inferences for objects that have been *introduced* (markers).
That Reflexivity does not hold in general has the important consequence that $\phi(a) \models\!\!\sim \neg\phi(a)$ is generally not an inconsistent statement. This is as it should be, for in PO–models $\neg\phi(\overline{a}_\phi) \leftrightarrow \forall x \neg\phi(x)$ is a valid formula. To derive an inconsistency we need $\phi(\overline{a}) \models\!\!\sim \phi(\overline{a})$. Without reflexivity for $\phi(\overline{a})$ there is no guarantee that $\mathcal{M}^* \models \psi(\overline{a}_\phi)$ will hold for *classical* consequences ψ of ϕ.

Introduction and Elimination of Connectives in the Identifiers

As we have introduction and elimination rules for the connectives on the level of $\mathcal{L}$–formulas, so we want to have rules introducing and eliminating these in the *identifiers* of objects. Addition of such rules to the standard proof system entails relating the *witnesses* for different $\mathcal{L}(x)$–formulas. We will only mention two candidate rules.

Conjunction

$$(\wedge \text{ II}) \ \frac{\psi(\overline{a}_\phi) \quad \xi(\overline{a}_\phi)}{\xi(\overline{a}_{\phi\wedge\psi})} \qquad (\wedge \text{ IE}) \ \frac{\psi(\overline{a}_\phi) \quad \xi(\overline{a}_{\phi\wedge\psi})}{\xi(\overline{a}_\phi)}$$

Here we have the rules of *Cautious Monotony* and *Cautious Cut* respectively (Makinson [11], Gabbay [5]). These rule will figure prominently in the next section.

3 Discourse Representation Theory

In our theory, candidates for discourse markers are constants indexed by formulas in one free variable (properties). A DRS is boolean combination of atomic sentences. Its markers are the constants $\overline{a}_\phi$ occurring in it, for which $\phi(\overline{a}_\phi)$ holds, i.e. constants that are indexed by the formulas that have led to their introduction. The flexibility of this set–up, can be illustrated by two examples

- Firstly, in the translation to logical formula, a choice will have to be made, whether to render phrases as *identifiers* of markers, or as *conditions* on them. So the sentence "A man walks" may give rise to $walk(a_{man})$ — translating "A man *walks*" — or to $man(a_{walk})$ — translating "A *man* walks" — or even to $man(a_{man \wedge walk})$ In this way, the topic–comment ('theme'–'rheme') distinction can be formalized in a natural way.
- Secondly, syntactic information can be used to determine the *logic* of the markers. For instance, the syntactic feature [± specified quantity] of plurals, may guide Existential Instantiation in the choice of a *logic domain*: a set of objects governed by rules of the extended proof system.

3.1 Basic Discourse Representation Structures

To start our formal discussion of DRT within the framework of partial object models, we restrict ourselves to *basic* DRS's, i.e., formulas $\psi(\overline{a}_\phi)$ where ϕ as well as ψ are *conjunctions of literals*. So, whenever we refer to $\mathcal{L}$–sentences, $\mathcal{L}(x)$–formulas or $\mathcal{A}_\mathcal{L}$ constants, we mean only elements from this restricted language. We can think of such sentences and formulas as *derived* from a purely first–order theory through the basic proof system.

Definition 5 Let S be a finite set of $\mathcal{L}$–sentences and $\mathcal{A}(S)$ the set of $\mathcal{A}_\mathcal{L}$–constants occurring in S, then

- the set $M(S)$ of *markers* of S is given by $M(S) = \{\overline{a}_\phi \in \mathcal{A}(S) \mid S \vdash \phi(\overline{a}_\phi)\}$
- S is *closed* if $\mathcal{A}(S) \cup |\mathcal{A}(S)| \subseteq M(S)$, i.e., for all $\overline{a}_\psi \in \mathcal{A}(S)$, $\overline{a}_\psi \in M(S)$ and if $\overline{a}_\psi \prec \overline{a}_\phi$ then $\overline{a}_\phi \in M(S)$

So, the markers of a set of sentences S, are the constants occurring in it that have been *introduced* ($\exists x \phi(x) \leftrightarrow \phi(\overline{a}_\phi)$); S is closed if all constants in S are markers.

Remark 2 The difference between the markers $M(S)$ of some set S of $\mathcal{L}$–sentences and the other constants occurring in elements of S can be interpreted in various ways. Here the markers are viewed as the constants that have been *introduced*, but other interpretations are: the constants that have been *constructed*, *declared* or *found* (relative to some search procedure).

Definition 6 Discourse Representation Structures
A *Discourse representation Structure* (DRS) is a finite set of $\mathcal{L}$–sentences. A DRS is *open* if it is not closed. A set S of $\mathcal{L}$–sentences is a *possible continuation* of a closed DRS D if $D \cup S$ is closed.

A continuation S of structure D need not itself be closed. The set $\Delta(S) = (\mathcal{A}(S) \cup |\mathcal{A}(S)|) - M(S)$ will be called the set of *anaphors* of S. Anaphors can occur up to any finite depth along the dependency relation: a pronoun like '*he*' is a top–level anaphor ($\in \mathcal{A}(S) - M(S)$), while a phrase like '*his* brother' will translate to $\overline{a}_{Brother(x,\overline{b})} \in M(S)$ where $b \notin M(S)$.
We will represent discourses $\phi(\overline{a}_\phi) \wedge \xi(\overline{a}_\chi)$ by $[(\overline{a}_\phi)\ (\xi(\overline{a}_\chi))]$ where the left element contains the *markers* and the right one the *conditions*.

Definition 7 Truth Definition for DRS's
$\mathcal{M}^* \models [(\overline{a}^1_{\phi_1} \ldots \overline{a}^n_{\phi_n})\ (\psi_1, \ldots, \psi_k)]$ iff

- for all markers $\overline{a}^i_{\phi_i} : \mathcal{M}^* \models \phi_i(\overline{a}^i_{\phi_i})$
- for all conditions $\psi_i : \mathcal{M}^* \models \psi_i$

Remember that we are only considering structures built from literals; a DRS may contain a second structure as a complex condition — of the form $\phi(\overline{a}_\phi) \wedge \psi(\overline{a}_\phi)$ — and such a condition will correspond to a DRS containing markers with *complex* indices.

3.2 Static and Dynamic DRT

A DRS may contain two kinds of constants. (1) Constants that have been introduced by an application of EI: for any two such constants $\langle \overline{a}, \phi(x) \rangle$, $\langle \overline{b}, \psi(x) \rangle$ we have $\overline{a} \neq \overline{b}$. (2) Constants $\langle \overline{a}, \phi(x) \rangle$, $\langle \overline{b}, \psi(x) \rangle$ where $\overline{a} = \overline{b}$. Here, at least one of them is not the product of an application of EI. Such a pair, we will interpret as referring to the same object: we will consider one of them as an *update* of the other. Given $\phi(\overline{a}_\phi)$, the condition $\psi(\overline{a}_\phi)$ gives *information* about $\overline{a}_\phi$ and updating the marker with this information results in the new marker $\overline{a}_{\phi\wedge\psi}$ where the *same constant* from $\mathcal{A}$ is indexed by the extended formula. We will introduce some proof rules to achieve this.
We will consider models for DRS's to be PO–models satisfying two additional principles, formulated as proof rules.

Definition 8 DRS–Models
Any model for a DRS will satisfy the rules of *Cautious Monotony* and *Cautious Cut* from the previous section.

Corollary The following rule is valid on all DRS models:

$$\text{(E–Rule)} \quad \frac{\psi(\overline{a}_\phi)}{\xi(\overline{a}_\phi) \leftrightarrow \xi(\overline{a}_{\phi\wedge\psi})}$$

This follows, by conditionalization, from C.M. and C.T. So if ψ holds for $\overline{a}_\phi$, then $\overline{a}_{\phi\wedge\psi}$ and $\overline{a}_\phi$ satisfy the same formulas, they denote the same partial object.

Definition 9 Markers $\overline{a}_\phi$, $\overline{a}_\psi$ are equivalent, $\overline{a}_\phi \sim \overline{a}_\psi$ if $\mathcal{M}^* \models \chi(\overline{a}_\phi) \iff \mathcal{M}^* \models \chi(\overline{a}_\psi)$ for all $\chi \in \mathcal{L}(x)$. Equivalence classes are defined accordingly, i.e., $[\overline{a}_\phi] = \{\overline{a}_\psi \mid \overline{a}_\phi \sim \overline{a}_\psi\}$

Lemma 1 Reduction to Objects
For every DRS, $[(\overline{a}_\phi)\ (\psi(\overline{a}_\phi))]$ and model $\mathcal{M}^*$ satisfying the E–Rule:

$$\text{if } \mathcal{M}^* \models [(\overline{a}_\phi)\ (\psi(\overline{a}_\phi))], \text{ then } \begin{cases} 1) & \mathcal{M}^* \models [(\overline{a}_{\phi\wedge\psi})\ (\emptyset)] \\ 2) & a_\phi \sim a_{\phi\wedge\psi} \end{cases}$$

The proof of this lemma is an immediate consequence of the E–rule. We have for instance

$$\frac{\phi(\overline{a}_\phi) \quad \psi(\overline{a}_\phi)}{\phi(\overline{a}_{\phi\wedge\psi}) \wedge \psi(\overline{a}_{\phi\wedge\psi})}$$

Notice that we derive a *reflexivity statement* $(\phi(\overline{a}_{\phi\wedge\psi}) \wedge \psi(\overline{a}_{\phi\wedge\psi}))$ without using EI.

Corollary Every closed discourse is reducable to a set of markers closed under successors along the dependency relation.

Note the key assumption that the DRS's are *closed*: an open DRS may be partially reducable, but it will always contain irreducable conditions.

This lemma will form the basis for our interpretation of Discourse Representation Structures. It entails that we can consider the *condition* $\psi(\overline{a}_\phi)$ of DRS $[(\overline{a}_\phi)\ (\psi(\overline{a}_\phi))]$ to be a map from $[(\overline{a}_\phi)\ (\emptyset)]$ to $[(\overline{a}_{\phi\wedge\psi})\ (\emptyset)]$ preserving all conditions that hold for $\overline{a}_\phi$. So from a general perspective, DRS conditions can be viewed as *update functions* of the sets of markers involved. Informally,

$$[\![\psi(\overline{a})]\!] : [(\overline{a}_\phi)\ (\emptyset)] \longmapsto [(\overline{a}_{\phi\wedge\psi})\ (\emptyset)]$$

and the relation between the traditional and the update perspective can be stated in a simple way:

Definition 10 Fixed Points
An equivalence class $[\overline{a}_\phi]$ of marker $\overline{a}_\phi$ is a *fixed point* for $\psi(\overline{a}_\phi)$ if $[\![\psi(\overline{a}_\phi)]\!]([\overline{a}_\phi]) = [[\![\psi(\overline{a}_\phi)]\!](\overline{a}_\phi)] = [\overline{a}_\phi]$.

Lemma 2 Update Lemma

$$\mathcal{M}^* \models [(\overline{a}_\phi)\ (\psi(\overline{a}_\phi))] \quad \text{iff} \quad [\overline{a}_\phi] \text{ is a fixed point of } [\![\psi(\overline{a}_\phi)]\!]$$

So, if $\psi(\overline{a}_\phi)$ already holds for marker $\overline{a}_\phi$ in $\mathcal{M}^*$, then updating only results in change of the *name* for the object a_ϕ.

In this way, we embed the standard, static, Discourse Representation Structures in a dynamic framework, where *conditions* are interpreted as *update* functions mapping (sets of) markers to (sets of) markers.

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