A Formal Comparison of Grids and Trees*

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Abstract

This paper presents a partial formalisation of phonological bracketed grid theory. It is subsequently argued that this formal object and the more well-known tree cannot be compared. Yet from the bracketed grid we can extract a construct called a superline. *The superline satisfies the Single Root Condition but not the Exclusivity Condition or the Non-Tangling Condition. Phonological X-bar structures are introduced and it is shown in what way they are different from both normal trees and bracketed grids. X-bar trees are a more restricted form of tree. They differ from bracketed grids and superlines in the restrictions imposed on (government) relations.*

1 Introduction

The rise of Non-linear Phonology shifted the phonologist's attention from the theory of rules (*The Sound Pattern of English*—Chomsky and Halle 1968) to the theory of representations. During the last decade phonologists have developed a theory of representations that is sufficiently rich and adequate to describe a wide range of facts from the phonologies of various languages.

It is a fairly recent development that these representations are being studied also from a purely formal point of view. There has been some work done on autosegmental structure (for instance Coleman and Local (1991) Bird and Ladd (1991), Bird and Klein (1990)) and also some work on metrical trees (like Coleman (1990) in unification phonology and Wheeler (1981), Moortgat and Morrill (1991) in categorial logics). As far as I know, apart from the pioneering work by Halle and Vergnaud (1987), hitherto no attention has been paid to the formal aspects of the most popular framework of metrical phonology nowadays, the *bracketed grids* framework (Halle and Vergnaud 1987, Hammond 1984, Hayes 1991, Kager 1989).

In this paper I study this representation from an algebraic point of view. The purpose of this is twofold. First, I want to define a formal language in which linguistic generalizations on metrical structure can be expressed. Secondly, I want to study some of the mathematical properties of this system. In particular I am interested in comparing this system with the formalism of trees and X-bar structures.

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The relation between bracketed grids and trees has been under discussion ever since the appearance of the former formalism. In Halle and Vergnaud (1987, henceforth HV), a whole section is dedicated to the subject (cf. section 5.5, pp. 161-170). Hammond (1984) independently argued for a formalism quite like bracketed grids, in which tree-like dominance lines are drawn between elements.

Other authors have also expressed the intuition that bracketed grids and tree structures (e.g. the [s(trong),w(eak)] labeled trees of Hayes (1981) and related work) are equivalent. An example is Van der Hulst (1991), writing: *Unless I miss something, it seems to me that the [bracketed grid notation] is equivalent to a tree notation.*

In this paper, I study Van der Hulst's intuition in some formal detail and show that it is wrong. The structure of the argument is as follows. In the second section of this paper, I build up a definition of the bracketed grid as a formal object. In section 3 I subsequently compare this formal object to the tree and in section 4 I compare both trees and grids to phonological X-bar structures. Section 5 is devoted to a conclusion.

2 The definition of a bracketed grid

HV themselves have devoted a major part of their book (part II) to pure formalization of the bracketed grid framework in terms of Horn Clause-like logics. Unfortunately there are a few problems with this formalization. I will mention three of them.

First, their formalization is not flexible enough to capture all instances of (bracketed) grid theory which are actually found in the literature of the last few years. They merely give a sketch of the specific implementation of bracketed grid theory as it is used in parts of their book. Modern work like Kager (1989) or Hayes (1991) cannot be described within this framework; and even with respect to their own empirical work, the formalization is incomplete, as we will see below.

Secondly, they use a lot of formal concepts which have no empirical or formal necessity like 'natural brackets' (cf. Blevins 1992).

Thirdly, their way of formalizing bracketed grids has very much a 'derivational' flavour. HV are more interested in how grids can be built than in what they look like. Although looking at the derivational aspects of a generative theory is an interesting and worthwile enterprise in itself, it makes HV's formalism less suitable for a comparison with metrical trees or other formal objects.

2.1 Lines and metrical lines

A *grid* in the linguistic literature is a set of lines, each line defining a certain subgroup of the stress bearing elements. Thus, in (1) (HV's (77), p.262), the elements (bot the asterisks – 'stars' – and the dots) on line 1 represent the syllables of the word *Tennessee*, the elements on line 2 represent the syllables with secondary stress and line 3 represents the syllable with primary stress. An element is drawn as a star *iff* it is also present at a higher level. Otherwise it is drawn as a dot:

```
(1)                .   line 3
      (.           *)  line 2
      (*     .)   (*)  line 1
      Ten    ne    see
```

We can formalize the notion of a line as follows:

Definition 1 (Line) *A line L_i is a pair $< A_i, \prec_i >$*
where $A_i = \{a_i^1, ..., a_i^n\}$; $a_i^1, \ldots, a_i^n$ constants, n *a fixed number*
$\prec_i$ is a total ordering on A_i which is transitive, asymmetric and irreflexive.
We say that $L_i \subset L_j$ if $A_i \subset A_j$; $\alpha \in L_i$ if $\alpha \in A_i$. Other set theoretic expressions are extended in a similar fashion.

Yet this formalisation is not complete for bracketed grids. It has to be supplemented by a theory about the brackets that appear on each line, i.e. by a theory of constituency and by a theory of what exactly counts as a star on a given line.

We have exactly one dot on top of each column of stars. Moreover, each *constituent* on a line has one star in it plus zero, one or more dots. The stars are heads. HV say that these heads *govern* their complements. This government relation can only be a relation which is defined in terms of precedence, because of the nature of a line. Suppose we make the *government* relation the primitive notion instead of the *constituent.* A metrical line is defined as a line plus a government relation on that line:

Definition 2 (Metrical line) *A metrical line ML_i is a pair $< L_i, R_i >$, where L_i is a line and R_i is a relation on L_i and an element of $\{\prec_i, \succ_i, \rightarrow_i, \leftarrow_i\}$*[1]

I assume that something like Government Requirement 1 holds, just as it is assumed in HV that every element is in a constituent:

Government Requirement 1 (to be revised below)
A line L_i meets the government requirement iff all dots on L_i are governed, i.e. a star is in a government relation to them.

Now a constituent can be defined as the domain that includes a star, plus all the dots hat are governed by this star. We have to be a little bit careful here, because we want to make sure that there is only one star in each constituent.

In order to ensure this, we adopt an idea from modern syntactic (GB) theory, viz. *Minimality*, which informally says that an element is only governed by another element if there is no closer governor. The definition of Phonological Minimality could look as follows:

Definition 3 (Minimality definition of phonological government)
$\alpha G_i \beta$ (α governs β on line i) iff α is a star and $\alpha R_i \beta \wedge \neg\exists\gamma, \gamma$ a star : $[\gamma R_i \beta \wedge \alpha R_i \gamma]$

We will give the formal definition of a star below, but first I can define the notion of a *constituent*:

[1] $\forall L_i : \forall \alpha\beta \in L_i$:

- $\alpha \succ_i \beta \stackrel{def}{=} \beta \prec_i \alpha$

- $\alpha \leftarrow_i \beta \stackrel{def}{=} \alpha \prec_i \beta \wedge \neg\exists\gamma : [\alpha \prec_i \gamma \wedge \gamma \prec_i \beta]$

- $\alpha \rightarrow_i \beta \stackrel{def}{=} \beta \leftarrow_i \alpha$

Actually, HV also use a fifth kind of constituent in their book, viz. one of the form (. * .). Because there has been a lot of criticism in the literature against this type of government (see e.g. Kager 1991 and Hayes 1991), I will not not discuss it here.

Definition 4 (Constituent) *A constituent on a line L_i is a set, consisting of exactly one star S in L_i plus all elements that are not stars but that are governed by S.*

Notice that in this formalism, the *constituent* is not a primitive. This means we cannot freely move around with brackets, as it is sometimes done in the literature. Even in HV (p. 188) we find a rule like the following, moving a bracket in order to analyse Odawa stress retraction:

(2) line 2: $\quad$ *) →)* /— #

This rule moves a bracket around a star, as if the bracket is a real element of the line. We can only do this if we consider a bracket to have substance, in other word if we view it as an *SPE*-like boundary symbol. This type of rule can not be expressed in the formalism presented here: in this formalism, the bracket is an epiphenomenon and we can not refer to it directly in our rules. However, the rule is against the spirit of HV's own formalisation as well, where brackets are seen as properties of constituents, not as independent elements (cf. the definition of the *Recoverability Condition* in HV (p. 10)).

2.2 Grids and bracketed grids

We now have a satisfying definition of a metrical line. We can define a grid as a collection of metrical lines, plus an ordering relation on them:

Definition 5 (Grid) *A grid G is a pair $<\mathcal{L}, \downarrow>$, where*
$\mathcal{L} = \{L_1, ..., L_n\}$, *$L_1, ..., L_n$ metrical lines*
$\downarrow$ is a total ordering on $\mathcal{L}$, such that
$\forall L_i, L_j \in \mathcal{L} : L_i \downarrow L_j \stackrel{def}{=} [L_i \subset L_j \wedge \forall \alpha, \beta \in L_i \cap L_j : [\alpha \prec_i \beta \Leftrightarrow \alpha \prec_j \beta]]$; *where $\subset$ denotes the* proper subset *relation.*

It is relatively easy to see that $\downarrow$ by this definition is transitive, asymmetric and irreflexive. We also define the inverse operator $\uparrow$ such that $L_i \uparrow L_j$ iff $L_j \downarrow L_i$.

The most interesting part of definition 5 is of course the $\downarrow$ ('above')-relation. Look once again at the grid in (1). Each of the lines in this grid has fewer elements than the one immediately below it. This follows from elementary pretheoretical reasoning. Every stressed syllable is a syllable, every syllable with primary stress also has secondary stress. I expressed this in definition 5 by stating that every line is a subset of the lines below it. The second part of the definition says that the relative ordering of the elements in each line is the same as that on the other lines.

We can also easily define the notion of a dot and a star, informally used in the definitions of *government* above. We can simply say that $\forall \alpha \in L_i : star_i(\alpha) \stackrel{def}{=} \exists L_j : [L_j \downarrow L_i \wedge (\alpha \in L_j)]$ and similarily, $\forall \alpha \in L_i : dot_i(\alpha)$ iff $\neg star_i(\alpha)$

The government requirement can now be fully formalised. It will subsequently be extended to the grid as a whole.

Government Requirement 2 (for lines — final version) *A metrical line L_i* meets the government requirement *iff* $\forall \alpha \in L_i : dot_i(\alpha) \Rightarrow \exists \beta \in L_i : [star_i(\beta) \wedge \beta G_i \alpha]$

We want two extra definitions on grids. Because all grids found in the linguistic literature are finite (i.e. they have a finite bottom line), I can define a *bottom line* and a *top line* for each grid:

Definition 6 (Top line and bottom line) *For a certain grid G, $\forall L_i \in G$*

- $(L_{top,G} = L_i) \stackrel{def}{=} \forall L_j \in G : [(L_i = L_j) \vee (L_i \downarrow L_j)]$
- $(L_{bottom,G} = L_i) \stackrel{def}{=} \forall L_j \in G : [(L_i = L_j) \vee (L_i \uparrow L_j)]$

Government Requirement 3 (for grids)
A grid G meets the government relation *iff all the lines $L_i \in G - \{L_{top,G}\}$ meet the government requirement*[2].

A last definition is needed here. If we look at the grids that are actually used in linguistic theory, it seems that there is always one line which has just one element. Furthermore, this line is the top line of the grid (the only line that could be above it would be an empty line, but that one doesn't seem to have any linguistic significance). Hayes (1991) also notes this where he states that *if prominence relations are obligatorily defined on all levels, then no matter how many grid levels there are, there will be a topmost level with just one grid mark.*

We can formalize this as follows[3]: A finite grid G which meets the government requirement is called a *complete grid* iff $|L_{top,G}| = 1$, i.e. $\exists\alpha : [\alpha \in L_{top,G} \wedge \forall\beta : [\beta \in L_{top,G} \Rightarrow \beta = \alpha]]$

I call this type of grid *complete* because one can easily construct a complete grid out of a given grid meeting the government requirement.

If $L_{top,G}$is non-empty, we construct a complete grid by projecting the rightmost (or alternatively the leftmost) element to a new line L_i and by adding the government requirement $\succ$ (or $\prec$) to $L_{top,G}$. Finally we add the relation $L_i \downarrow L_{top,G}$ to the grid, i.e. we make L_i to the new $L_{top,G}$.

If the top line of the grid is empty, we remove this line from the grid and proceed as above. If the top line has exaclty 1 element, the grid already is complete an we can stop.

3 Grids and trees

In this section, I show to what extent bracketed grids and trees are really different formal systems. First recall the standard definition of a tree (I cite from Partee, Ter Meulen and Wall 1990)[4]:

[2]We have to say something special with regard to $L_{top,G}$, because on this line all elements by definition are ungoverned dots. Hence they can never satisfy the government requirement.

[3]The reader should keep in mind that the type of grid presented here is a very strict one. This is done on purpose, because this is the grid that comes closest to a tree. In modern metrical work one makes use of a device called *Weak Local Parsing*, which keeps part of the structure 'unparsed', i.e. it allows for ungoverned dots on a limited scale. I could certainly define a *WLP*-bracketed grid in my formal language, but it would be immediately clear that this type of grid is different from a tree, because trees are by definition connected graphs.

[4]For the moment, I will not consider Q and L, because these are relatively unimportant for my present aim and goal and there is nothing comparable to the labeling function in my definition of bracketed grids. For now I will study *unlabeled* trees. Notice however that the trees actually used in the phonological literature do use at least a binary set of labels {s, w}. We will also see that X-bar structures make crucial use of labeling.

Definition 7 *A (constituent structure) tree is a mathematical configuration* $<N, Q, D, P, L>$, *where*
N is a finite set, the set of nodes
Q is a finite set, the set of labels
D is a weak partial order in $N \times N$, the dominance relation
P is a strict partial order in $N \times N$, the precedence relation
L is a function from N into Q, the labeling function
and such that the following conditions hold:

(a) $\exists \alpha \in N : \forall \beta \in N : [<\alpha, \beta> \in D]$ *(Single root condition)*
(b) $\forall \alpha, \beta \in N : [(<\alpha, \beta> \in P \vee <\beta, \alpha> \in P) \Leftrightarrow (<\alpha, \beta> \notin D \wedge <\beta, \alpha> \notin D)]$ *(Exclusivity condition)*
(c) $\forall \alpha, \beta, \gamma, \delta : [(<\alpha, \beta> \in P \wedge <\alpha, \gamma> \in D \wedge <\beta, \delta> \in D) \Rightarrow <\gamma, \delta> \in P]$ *(Nontangling condition)*

It is clear that bracketed grids and trees have structures which cannot be compared immediately. Bracketed grids are pairs consisting of a set of complex objects (the lines) and one total ordering relation defined on those objects (the above relation). Trees on the other hand are sets of simple objects (the nodes) with two relations defined on them (dominance and precedence). These simply appear to be two different algebras where no isomorphism can be defined.

Yet if we decompose the algebraic structure of the lines, we see that there we have sets of simple objects (the elements of the line) plus two relations defined on them. One of those relations ($\prec_i$) is a strict partial order, just like P. The other relation, G_i, is vaguely reminiscent of dominance.

A line is not a tree, however. Although $\prec_i$ has the right properties, it is not so clear that G_i does. While this relation clearly is asymmetric (because it is directional), it is not a partial order because it is not transitive and because it is irreflexive.

A more interesting relation emerges if we consider the grid as a whole. Because trees are finite structures, we need to consider *finite grids* only. The line $L_{bottom,G}$has the property that $\forall \alpha \forall L_i \in G : [\alpha \in L_i \Rightarrow \alpha \in L_{bottom,G}]$. This follows from the definitions of $L_{bottom,G}$ and the 'above' relation.

This means that all basic elements of the grid are present on $L_{bottom,G}$and, as we have seen above, we can equal P to $\prec_{bottom,G}$. Furthermore, we can build up a 'super-government' relation $\mathcal{G}$, which we define as the disjunction of all government relations G_i in G:

Definition 8 (Supergovernment relation of a grid G)
$$\mathcal{G}_G \stackrel{def}{=} \bigcup_{L_i \in G} \{<\alpha, \beta> \mid \alpha G_i \beta \wedge dot_i(\beta)\}$$

If we want to compare $\mathcal{G}$ to dominance, we have to make sure it is a partial order. However, $\mathcal{G}$ obviously still is irreflexiv and it also is intransitive. For this reason, I take the transitive and reflexive closure of $\mathcal{G}$, which I call $\mathcal{G}^*$.

Using this relation, we can define the superline of a linguistic grid:

Definition 9 (Superline)
The superline SL *of a linguistic grid G is the tuple* $< A_{bottom,G}, \prec_{bottom,G}, \mathcal{G}^*_G >$.

The superline is an entity which we can formally compare to a tree, with $\prec_{bottom,G} = P$, $A_{bottom,G} = N$, $\mathcal{G}^* = D$. Of most interest are the complete linguistic grids, firstly because these are the ones that seem to have most applications in linguistc theory and secondly because the requirement that they be *complete* (i.e. their $L_{top,G}$ should have exactly one element) mirrors the single root condition on trees. From now on, I will use the abbreviation CLG for 'complete linguistic grid'.

Note that I also restrict my attention to grids which meet the government requirement, i.e. to *linguistic* grids. I am not so sure that this restriction is equally well supported by metrical theory as the restriction to completeness. However, the restriction to linguistic grids makes sure that all elements in the grid participate in the government relation, because everything ends as a star somewhere and hence has to be governed by another element.

We can now prove that for every linguistic grid G, if G is complete, then $\mathcal{S}L_G$ satisfies the Single Root Condition.

The formal proof of this theorem is given in Van Oostendorp (1993). Hopefully the intuition behind it is clear: the requirement that a complete linguistic grid has only one element in the top-most line is equal to the requirement that a tree has one and only one root node.

So superlines have one important characteristic of trees. Yet *exclusivity* and *non-tangling* still do not hold for superlines of CLGs, even if they meet the government requirement. The reason why these conditions do not hold is that, on lines as well as on superlines, elements can both govern and precede another element. Exclusivity and nontangling are meant to keep precedence and dominance apart.

As a matter of course the differences between trees and bracketed grids become much less dramatic if we look at more restricted types of trees. Van Oostendorp (1993) shows that one restricted type of tree can be found which is exactly isomorphic to bracketed grids. This is the tree structure that is used in Dependency Phonology, which also do not obey Exclusivity and Non-Tangling..

4 X-bar trees

Maxwell (1992) contends that the differences between Dependency Graphs and X-bar structures are minimal. It remains to be shown whether there are any formal differences between the bracketed grids that are presented in this paper and the 'X-bar structures in lines' as they are represented in Levin (1985), Rubach and Booij (1990) and Hermans (1991), among others.

X-bar structures have never been given a full algebraic formalisation and comparison with trees, not even within mathematical syntax, even though they are among the most widely used formalisms at the moment. For this reason, I will give a brief characterisation of a general X-bar schema below. According to this schema, an X-bar tree is a specialized kind of tree.

Definition 10 (Simple X-bar tree)
An X-bar tree is a constituent structure tree $<N, Q, D, P, L>$ *such that*

1. $Q = QB_0 \cup \ldots \cup QB_n$, *where* $QB_0, \ldots, QB_n$ *are pairwise disjoint sets, referring to bar levels (generally, n is taken to be 2)*

2. $Q = QC_1 \cup \ldots \cup QC_m$, *where* $QC_1, \ldots, QC_m$ *are pairwise disjoint sets, referring to categories*
3. $\forall\alpha \in N : [\forall\beta \in N : [<\alpha, \beta> \in D] \Rightarrow L(\alpha) \in QB_n]$
4. $\forall\alpha \in N : [L(\alpha) \in QB_0 \Leftrightarrow \neg\exists\beta \in N : [\alpha M \beta]]$ *(where M 'mother of' is immediate dominance)*
5. $\forall\alpha \in N \forall\beta \in N : [\alpha \approx \beta \stackrel{def}{=} [\alpha M \beta \in D \wedge L(\alpha) \in QB_i \wedge L(\beta) \in QB_j \wedge j \leq i \wedge L(\alpha) \in QC_x \wedge L(\beta) \in QC_x]]$
6. $\forall\alpha \in N : [\exists\beta \in N : [\alpha M \beta] \Rightarrow \exists\gamma \in N : [\alpha \approx \gamma]]$
7. $\forall\alpha \in N \forall\beta \in N \forall\gamma \in N : [\alpha \approx \beta \wedge \alpha M \gamma \wedge \beta \neq \gamma] \Rightarrow [\neg(\gamma \approx \beta) \wedge (L(\gamma) \in QB_n)]$

I will briefly go through the items of this definition one by one. 10.1 divides the set of labels into a certain number of disjoint subsets, each subset defining a certain bar-level. For instance, Q might be the set of labels {N^0, A^0, P^0, V^0, $\overline{N}$, $\overline{A}$, $\overline{P}$, $\overline{V}$, $\overline{\overline{N}}$, $\overline{\overline{A}}$, $\overline{\overline{P}}$, $\overline{\overline{V}}$}. In that case QB_0 could be {N^0, A^0, P^0, V^0} and QB_1 could be {$\overline{N}$, $\overline{A}$, $\overline{P}$, $\overline{V}$}.

Similarily, according to 10.2, the set of labels can be divided into a certain number of disjoint subsets, defining categories. In the sample Q just given we could for instance suppose that QC_1 = {N^0, $\overline{N}$, $\overline{\overline{N}}$} and QC_2 = {A^0, $\overline{A}$, $\overline{\overline{A}}$}. We could of course also extend the set of labels in order to include functional and other heads.

It has been proposed both for bar levels and for categories that we should work with binary features instead of integers or unanalysable category names. For instance, Muysken (1982) proposes that bar levels should be analyses by using the features [±max]/[±proj] and it is more or less standard to analyse the category labels as [±V, ±N]. From an algebraic or set theoretic point of view, these issues are immaterial, however, because the features still define disjoint sets

10.3 says that the root of an X-bar tree should always be a maximal projection. Notice that this causes an important difference between X-bar trees and 'normal' trees: while a branch of a tree is always a tree, it is not necessarily true that a branch of an X-bar tree is always an X-bar tree — even though it is still a tree.

10.4 says that all the leaves of the X-bar tree are of a certain category and that all elements of this category are leaves[5]. Again, this distinguishes X-bar trees from normal trees: if we cut a branch from a normal tree, the result is again a tree. If we cut a branch from an X-bar tree the result is not always again an X-bar tree (but again, it *is* a tree).

10.5 defines an important relation in X-bar trees, viz. 'is the projection of'. The version presented here allows recursion of all bar levels, so $\overline{X}$ can head an $\overline{X}$ and $\overline{\overline{X}}$ can head an $\overline{\overline{X}}$, but it could easily be redefined to exclude such recursion: in that case we say $j < i$ instead of $j \leq i$. 10.6 conseqently says that every element has a head, except if it is a leave and 10.7 says that no element has more than one head and that all sisters to a head are maximal projections.

The latter two statements of course define the core of X-bar theory, as used in GB syntax[6].

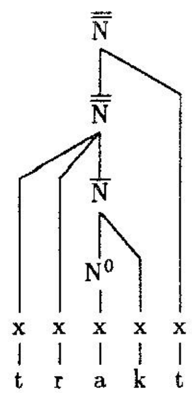
In the phonological literature the structures that are actually used are somewhat more limited. I now turn my attention to *phonological X-bar trees*, which differ from

[5]At first sight this might seems to exclude recursion at the X^0 level. Because we can always relabel a tree in such a way that it conforms to 10.4, this is a matter of no great importance.

[6]Specific versions of syntactic X-bar theory sometimes have additional constraints, like a binary branching requirement, but I will not go into these.

the schema in definition 10 in certain ways. I will take Levin (1985) as my guideline, because it is the most accessible and extensive presentation of phonological X-bar theory available at the moment. This theory is most commonly used to represent syllable structure. The English syllable *tract* is represented in this theory as follows:

(3)



I will discuss the steps of the definition one by one:

10.1 Like in syntactic X-bar theory, it is assumed that the lowest X-bar level is X^0 and the highest level is X^2. We should be a little bit careful here however. As one can see in (3), we have an intermediate level of x-slots between the bar levels of N and the segmental material. These skeletal points play a crucial role in all phonological phrase structure descriptions, and therefore they should be considered part of phonological X-bar theory. This gives us *four* bar levels: $\{\text{x}, \text{N}^0, \overline{\text{N}}, \overline{\overline{\text{N}}}\ \}$

10.2 As I stated above, most work in phonological X-bar theory is concerned with syllable structure. This implies that there is often just one category: N(ucleus).

10.3 This axiom also holds for phonological X-bar structures. An X-bar structure is not finished until it reaches its highest level.

10.4 Also this is true for phonological X-bar structures, at least if we keep to the assumption just made that x's are the real end-points of X-bar structures. These do not dominate anything although they are associated to autosegmental material in the sense of autosegmental phonology. The relation between the first x and the /t/ in (3) is different from that between x and N: the /t/ is not a daughter of the x-slot, but it is associated to it, i.e. they overlap in time (Sagey 1986).

10.5 is a definition which trivially holds on any X-bar structure. Notice however that we need recursion on the $\overline{\overline{\text{N}}}$, but not on the $\overline{\text{N}}$ or N^0. An adequate definition of a phonological X-bar theory would take this into account by making 10.5 somewhat more precise.

10.6 is true, if we assume that N^0 counts as a projection of the x dominated by it.

10.7 gives the most important difference between phonological and syntactic X-bar trees: in phonological X-bar trees, the sisters of the head are never maximal projections; to the contrary, they are always *minimal* elements. namely x's. A given N^0 always has x-slots as its non-head daughters, never full syllables.

Taking these considerations into account, we can give the following definition of a phonological X-bar tree:

Definition 11 (Phonological X-bar tree)
A phonological X-bar tree is a tree $<N, Q, D, P, L>$ *such that*

1. $Q = QB_0 \cup \ldots \cup QB_3$, *where* $QB_0, \ldots, QB_3$ *are pairwise disjoint sets, referring to bar levels* ($QB_0 = \{x\}$, $QB_1 = \{N^0\}$, $QB_2 = \{\overline{N}\}$, $QB_3 = \{\overline{\overline{N}}\}$)
2. $Q = QC_1 = \{x, N^0, \overline{N}, \overline{\overline{N}}\}$
3. $\forall \alpha \in N : [\forall \beta \in N : [<\alpha, \beta> \in D] \Rightarrow L(\alpha) \in QB_n$
4. $\forall \alpha \in N : [L(\alpha) \in QB_0 \Leftrightarrow \neg\exists\beta \in N : [\alpha M \beta]]$
5. $\forall \alpha \in N \forall \beta \in N : [\alpha \approx \beta \stackrel{def}{=} [\alpha M \beta \in D \wedge L(\alpha) \in QB_i \wedge L(\beta) \in QB_j \wedge (j < i \vee j = i = 3)]]$
6. $\forall \alpha \in N : [\exists \beta \in N : [\alpha M \beta] \Rightarrow \exists \gamma \in N : [\alpha \approx \gamma]]$
7. $\forall \alpha \in N \forall \beta \in N \forall \gamma \in N : [\alpha \approx \beta \wedge \alpha M \gamma \wedge \beta \neq \gamma] \Rightarrow [\neg(\gamma \approx \beta) \wedge (L(\gamma) \in QB_0)]$

I already discussed the differences between these X-bar structures and trees. A word remains to be said about the difference between X-bar structures and bracketed grids.

The first three clauses in (11) all refer to labels; an X-bar tree by definition cannot be an unlabeled tree, whereas bracketed grids are always unlabeled. This seems to cause already one important difference between the two formalisms.

This is certainly true for the categorial label, to which there is no real alternative in bracketed grid theory. Notice, however, that it is assumed that there is just one set QC which includes all members of Q. The categorial labels thus do not play a very important role in the system at all.

The bar level sets, on the other hand, can be mimicked in the bracketed grid system quite easily: we can say that an element ϵ of a given bracketed grid Γ has the label X^n iff ϵ is an element of line L_i in Γ and there is no L_j, such that $L_j \downarrow L_i$ and $\epsilon \in L_j$. In other words, a bar level set is a line.

We have already seen that the Single Root Condition holds for (superlines of) complete grids. According to 11.3, the root, i.e. the element that (transitively) governs all other elements is an element of $L_{top,G}$. This is trivially true: if the root would not be a member of $L_{top,G}$, it would be transitively governed by a member μ of $L_{top,G}$, distinct from it. But then a contradiction would arise: μ would not be governed by the root because of the asymmetry of the government relation, hence the root would not be the element governing all other elements.

11.4 is equally trivial: it says that there are no elements below the bottom line.

The projection relation $\alpha \approx \beta$ defined in 11.5 can not be interpreted directly in the mapping, proposed here, because $\alpha \approx \beta$ seems to hold of every pair of elements $<\alpha, \beta>$ which are on different lines, or both on $L_{top,G}$. More generally we cannot really define 'is a projection of' in a bracketed grid: the element and the projection of that element are indistinguishable, so we would have to say $\alpha = \beta$ in stead of $\alpha \approx \beta$.

Because 11.6 and 11.7 are stated in terms of 11.5, they themselves now become *stricto sensu* meaningless. However, the spirit of 11.6 can still be captured by the formula $\forall \alpha \in G : \exists \beta \in G[\alpha M \beta \wedge \beta \in L_i \wedge \alpha \in L_j \wedge L_j \downarrow L_i] \Rightarrow [\alpha \in L_i]$, which is trivially true, because $\forall \alpha \in G : [\alpha \in L_j \wedge L_j \downarrow L_i \Rightarrow \alpha \in L_i]$ is a theorem in bracketed grid theory[7].

11.7, however, shows us a final and irrepairable difference between phonological X-bar trees and bracketed grid structures. Translated into bracketed grid terms, this clause says that all governed elements of a grid are part of $L_{bottom,G}$. The tree in (3) could consequently only be translated into the following grid:

[7] This is the so-called *Complex Column Constraint* of Hayes 1991; for a proof that the CCC is a theorem in bracketed grid theory see Van Oostendorp 1993.

(4)
```
          .          L_top,G
(.  .  *  .  .)      L_bottom,G
 t  r  a  k  t
```

The necessary government relation in $L_{bottom,G}$ can however in no way be expressed in the system presented in this article or in the formalism of HV. Syntactic trees, on the other hand can also not be translated into grids following this algorithm without loding essential information. Because all governed elements in syntactic X-bar trees are maximal projections, all elements would end up as elements of $L_{top,G}$. This means that syntactic X-bar trees would always end up as grids of just one line[8].

5 Conclusion

Let me summarize the results of this paper. We have seen that from bracketed grids we can extract *superlines*, on which the government relations of the normal lines are conflated.

These superlines are equivalent to some sort of unlabeled trees, under a very weak definition of the latter notion. Whereas the minimal restrictions of the Single Root Condition do hold, the same is not necessarily true for the Exclusivity Condition and the Non-Tangling Condition. Superlines do correspond to a very specialized sort of tree, the *Dependency Graphs* of Dependency Phonology.

Finally, it is shown that the X-bar structures that are used in some versions of syllable theory have properties that distinguish them from both plain trees and bracketed grids. In particular, the notion of *government* as defined on an X-bar tree cannot be translated to its counterpart on a bracketed grid.

It is a common practice, both in theoretical and in computational linguistics, to claim that most so-called linguistic theories are actually 'notational variants'. If we take the term 'α is a notational variant of β' to mean that we can define a homomorphism between α and β, we have shown that although three of the most popular devices of representing metrical structure, trees, grids and X-bar structures are very similar, they are *not* notational variants.

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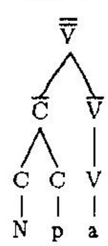
[8] Michaels (1992) offers a very interesting view on phonological X-bar structure from our persective. He uses trees like the following for representing syllable structure:

(i)



In this tree, each daughter of a node has the same bar level; furthermore, there is no recursion. This makes the X-bar tree much more similar a bracketed grid. Notice, however that Michaels uses two different types of category, C and V in this kind of tree. It is not clear to me from the article how relevant this distinction is.

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